

ITP-Budapest 564
hep-ph/0103093

EFFECT OF THE SCALAR CONDENSATE ON THE LINEAR GAUGE FIELD RESPONSE IN THE ABELIAN HIGGS MODEL

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ABSTRACT

The effective equations of motion for low-frequency mean gauge fields in the Abelian Higgs model are investigated in the presence of a scalar condensate, near the high temperature equilibrium. We determine the current induced by an inhomogeneous background gauge field in the linear response approximation up to $\mathcal{O}(e^4)$, assuming adiabatic variation of the scalar fields. The physical degrees of freedom are found and a physical gauge choice for the numerical study of the combined Higgs+gauge evolution is proposed.

CERN-TH/2001-074
ITP-BUDAPEST 564
March 2001

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1 Introduction

The real-time transformation of light gauge fields into massive intermediate vector bosons is a subject of increasing cosmological interest.

Recently, Garcia-Bellido *et al.* [1] (see also [2]) assumed that the coupled inflaton+Higgs system was far out of equilibrium when the energy density of the expanding Universe has passed the point corresponding to the thermal Higgs transition. With the supplementary condition that the reheating temperature after the exit from the inflationary period did not exceed the electroweak critical temperature, they demonstrated in a (1+1)-dimensional toy model, that large enough matter–antimatter asymmetry could have survived till today. The existence of non-equilibrium Higgs transitions in 3+1 dimensions has been demonstrated by Rajantie *et al.* [3]. In these investigations the classical mean field equations were used with renormalized and temperature-dependent couplings.

A more traditional field of the investigation is the evolution of the baryon asymmetry through the electroweak phase transition, accompanying the onset of the Higgs effect. It has been thoroughly discussed, under the assumption that the system is in thermal equilibrium, with specific emphasis on the high-temperature sphaleron rate [4, 5, 6]. Near equilibrium, the gauge-invariant HTL action dominates the influence of the thermalized quantum fluctuations on the motion of the mean fields with $k \simeq \mathcal{O}(gT)$ [7, 8, 9]. This term was taken into account in real-time simulations of the sphaleron rate [5] (see also [10]).

The framework for the theoretical study of the baryon asymmetry has been somewhat modified with the realization that, within the Standard Model, the Higgs effect sets in via smooth phase transformation, characterized by an analytic variation of the order parameter [11, 12, 13, 14]. Under such circumstances the expectation value of the Higgs field is different from zero also above the electroweak energy scale. In this context it is remarkable that the leading HTL correction is insensitive to the presence of the scalar condensate [15, 5].

In this note we go one step beyond the derivation of the HTL correction. Our aim is to find the leading effect due to the presence of an arbitrary constant background field: $\overline{\Phi}$. In equilibrium, such a background is sufficient for the calculation of the effective potential, but not the full effective action. Similarly here, we restrict somewhat the generality of our correction terms to the equations of motion, namely the corrections to the Φ^2 – A^2 vertex will be determined neglecting the non-local effects in the scalar field.

The high-temperature limit of the resulting expression of the induced current shows that the next-to-leading “mass and vertex” corrections can be uniquely decomposed into gauge-fixing invariant and gauge-fixing dependent modes. In this way we can propose an expression of wider applicability for the Landau damping effect in the presence of a scalar condensate.

Baacke and his collaborators [16, 17, 18] have applied, in a series of papers, a complementary approach to the Higgs effective action without assuming the presence of a general gauge background.

A non-trivial mean gauge field configuration reacts back on the scalar condensate. Our final goal is to derive a coupled set of equations for the scalar condensate and the mean gauge field, which is appropriate for studying the real-time onset of the Higgs effect and the variation of the sphaleron generation and decay rate. We propose a physical gauge for the numerical solution of these equations where the gauge-fixing dependent mode does not propagate.

The generalization of the derivation to non-Abelian systems presents only technical complications. The Abelian calculation has been performed both in the standard real-time generator functional formulation [19] and with an iterated solution of the linearized Heisenberg equations of the quantum fluctuations [20, 21, 22]. We shall use below the latter approach, where the adequate two-point functions enter more naturally. The general linear response theory will be described in a separate publication [23].

2 Mean Field Equations in the Abelian Higgs model

The Lagrangian of the model in the $O(2)$ notation is the following:

$$\mathcal{L} = -\frac{1}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} + \frac{1}{2}(\partial_\mu\hat{\Phi})^2 - \frac{1}{2}m^2\hat{\Phi}^2 + e\hat{A}_\mu\hat{J}^\mu + \frac{e^2}{2}\hat{A}^2\hat{\Phi}^2 - \frac{\lambda}{24}(\hat{\Phi}^2)^2, \quad (1)$$

where $\hat{\Phi} = (\hat{\Phi}_1, \hat{\Phi}_2)$ and $\hat{J}_\mu = \hat{\Phi}_2\partial_\mu\hat{\Phi}_1 - \hat{\Phi}_1\partial_\mu\hat{\Phi}_2$. We split the fields into a mean field (A_μ, Φ) and a fluctuation contribution

$$\hat{A}_\mu = A_\mu + a_\mu, \quad \hat{\Phi} = \Phi + \varphi, \quad \langle a_\mu \rangle = \langle \varphi \rangle = 0 \quad (2)$$

at any time (averaging is understood with respect to the initial density matrix). We assume that the scalar background is constant and that it points to the Φ_1 direction, its value being $\bar{\Phi}$. We use in the sequel the notations $m_W = e\bar{\Phi}$, $m_H^2 = m^2 + \lambda\bar{\Phi}^2/2$, $m_G^2 = m^2 + \lambda\bar{\Phi}^2/6$, although these are not the vacuum values.

We fix a 't Hooft R_ξ gauge by changing the Lagrangian as

$$\mathcal{L} \rightarrow \mathcal{L} - \frac{1}{2\xi}(\partial_\mu\hat{A}^\mu + \xi m_W\varphi_2)^2 - \bar{c}(\partial^2 + \xi m_W^2 + e\xi m_W\varphi_1)c, \quad (3)$$

where c is the ghost field. In this way the field φ_2 receives an additional mass contribution ξm_W^2 .

The operatorial equation of motion (EOM) separately induces EOMs for the mean fields and the fluctuations. The average of the operatorial EOM for the gauge field reads as

$$\left[(\partial^2 + m_W^2)g_{\mu\nu} - \left(1 - \frac{1}{\xi}\right)\partial_\mu\partial_\nu \right] A^\nu + j_\mu^{ind}(A) = 0, \quad (4)$$

with the induced current

$$j_\mu^{ind}(A) = e\langle j_\mu \rangle + 2e^2\bar{\Phi}\langle a_\mu\varphi_1 \rangle + e^2A_\mu\langle \varphi^2 \rangle + e^2\langle a_\mu\varphi^2 \rangle, \quad (5)$$

where $j_\mu = \varphi_2\partial_\mu\varphi_1 - \varphi_1\partial_\mu\varphi_2$. We will perform the calculations at the one-loop level, when the last term does not contribute. The subtraction of (4) from the full equation yields the EOM for the fluctuations. At one loop it is sufficient to consider only the equations linearized in the fluctuations. On the other hand, since we want to calculate j_μ^{ind} in the linear response

approximation, we can neglect all terms non-linear in A . These assumptions make the equations very simple:

$$\begin{aligned} & \left[-(\partial^2 + m_W^2)g_{\mu\nu} + \left(1 - \frac{1}{\xi}\right) \partial_\mu \partial_\nu \right] a^\nu - 2e^2 \bar{\Phi} A_\mu \varphi_1 = 0, \\ & (\partial^2 + m_H^2)\varphi_1 + 2eA^\mu \partial_\mu \varphi_2 + e(\partial A)\varphi_2 - 2e^2 \bar{\Phi} A^\mu a_\mu = 0, \\ & (\partial^2 + m_G^2 + \xi m_W^2)\varphi_2 - 2eA^\mu \partial_\mu \varphi_1 - e(\partial A)\varphi_1 = 0. \end{aligned} \quad (6)$$

The ghost fields follow a free EOM, so that they do not influence the present calculation. These equations are solved to linear order in the A background:

$$\begin{aligned} a_\mu(x) &= a_\mu^{(0)}(x) - 2e^2 \bar{\Phi} \int d^4 z G_{\mu\nu}^R(x-z) A^\nu(z) \varphi_1^{(0)}(z), \\ \varphi_1(x) &= \varphi_1^{(0)}(x) + e \int d^4 z G_1^R(x-z) \left[2A^\mu(z) \partial_\mu \varphi_2^{(0)}(z) + (\partial A)(z) \varphi_2^{(0)}(z) - 2e \bar{\Phi} A^\mu(z) a_\mu^{(0)}(z) \right], \\ \varphi_2(x) &= \varphi_2^{(0)}(x) - e \int d^4 z G_2^R(x-z) \left[2A^\mu(z) \partial_\mu \varphi_1^{(0)}(z) + (\partial A)(z) \varphi_1^{(0)}(z) \right], \end{aligned} \quad (7)$$

where the superscript zero denotes the solutions of the free EOM, and G^R 's are the free retarded Green functions⁴.

For the induced current (5) we need the expectation values of certain local products of the fluctuating fields. For example, it directly follows from the above equations (with $\langle AB \rangle_0 \equiv \langle A^{(0)} B^{(0)} \rangle$), that

$$\begin{aligned} \langle \varphi_2(x) \partial_\mu \varphi_1(x) \rangle &= e \int d^4 z \left\{ \partial_\mu^x G_1^R(x-z) \left[2A^\nu(z) \langle \varphi_2(x) \partial_\nu^z \varphi_2(z) \rangle_0 + (\partial A)(z) \langle \varphi_2(x) \varphi_2(z) \rangle_0 \right] \right. \\ &\quad \left. - G_2^R(x-z) \left[2A^\nu(z) \langle \partial_\mu \varphi_1(x) \partial_\nu \varphi_1(z) \rangle_0 + (\partial A)(z) \langle \partial_\mu \varphi_1(x) \varphi_1(z) \rangle_0 \right] \right\}. \end{aligned} \quad (8)$$

We define the local products in a symmetric way (i.e. $\frac{1}{2} \langle \varphi_2(x) \partial_\mu \varphi_1(x) + \partial_\mu \varphi_1(x) \varphi_2(x) \rangle$) and introduce

$$\Delta(x-z) = \frac{1}{2} \langle \varphi(x) \varphi(z) + \varphi(z) \varphi(x) \rangle_0. \quad (9)$$

Then the above expression can be written in Fourier space as

$$\langle \varphi_2 \partial_\mu \varphi_1 \rangle(Q) = -e A^\nu(Q) \int \frac{d^4 p}{(2\pi)^4} p_\mu (2p-Q)_\nu \left[G_1^R(p) \Delta_2(Q-p) + G_2^R(Q-p) \Delta_1(p) \right]. \quad (10)$$

The evaluation of other expectation values goes along the same line, finally giving

$$\begin{aligned} e \langle j_\mu \rangle(Q) &= -e^2 A^\nu(Q) \int \frac{d^4 p}{(2\pi)^4} (2p-Q)_\mu (2p-Q)_\nu \left[G_1^R(p) \Delta_2(Q-p) + G_2^R(Q-p) \Delta_1(p) \right], \\ 2e^2 \bar{\Phi} \langle a_\mu \varphi_1 \rangle(Q) &= -4e^2 m_W^2 A^\nu(Q) \int \frac{d^4 p}{(2\pi)^4} \left[G_{\mu\nu}^R(p) \Delta_1(Q-p) + G_1^R(Q-p) \Delta_{\mu\nu}(p) \right], \\ e^2 A_\mu \langle \varphi^2 \rangle &= e^2 A_\mu(Q) \int \frac{d^4 p}{(2\pi)^4} [\Delta_1(p) + \Delta_2(p)]. \end{aligned} \quad (11)$$

⁴Here the definition $K G^R = -\delta$ is used, where K denotes the free kernel.

Assuming that the free fluctuations are in thermal equilibrium, the propagators can be related to the corresponding spectral functions [24], which are the discontinuities of the free kernels of eq. (6). Introducing

$$\Delta_{m^2}(p) = 2\pi \left(\frac{1}{2} + n(|p_0|) \right) \delta(p^2 - m^2), \quad \text{and} \quad G_{m^2}^R(p) = \frac{1}{p^2 - m^2 + i\varepsilon p_0}, \quad (12)$$

where n is the Bose–Einstein distribution, we have

$$\Delta_1 = \Delta_{m_H^2}, \quad \Delta_2 = \Delta_{m_G^2 + \xi m_W^2}, \quad \Delta_{\mu\nu} = -g_{\mu\nu} \Delta_{m_W^2} + \frac{p_\mu p_\nu}{m_W^2} (\Delta_{m_W^2} - \Delta_{\xi m_W^2}), \quad (13)$$

and analogously for the corresponding G^R 's.

3 High-temperature expansion

The leading HTL term of j_μ^{ind} in the high-temperature expansion comes from $e \langle j_\mu \rangle$ by neglecting all the masses [15]. Our aim is to calculate the first subleading term proportional to Φ^2 instead of T^2 in the high-temperature expansion.

We emphasize that $j^{ind}(Q) = j^{ind}(-Q)^*$, therefore all partial contributions that are odd under hermitian Q -reflection can be freely omitted.

A useful formula, which simplifies the calculations, is the following [25]:

$$\begin{aligned} I_F(Q, m^2, M^2) &= \int \frac{d^4 p}{(2\pi)^4} F(Q, p) \left(G_{m^2}^R(p) \Delta_{M^2}(Q - p) + G_{M^2}^R(Q - p) \Delta_{m^2}(p) \right) \\ &= \int \frac{d^4 p}{(2\pi)^4} F(Q, \frac{Q}{2} - p) \frac{\Delta_{m^2}(p - \frac{Q}{2}) - \Delta_{M^2}(p + \frac{Q}{2})}{2pQ + m^2 - M^2}. \end{aligned} \quad (14)$$

The tensorial structures appearing in (11) when cast into the form (14) imply the appearance in F of the p -dependent terms $p_\mu p_\nu$, $p_\mu Q_\nu + p_\nu Q_\mu$ and also of terms independent of p .

3.1 Even terms

We start the discussion with the terms even under p -reflection and introduce the notation $f(p) = (F(Q, Q/2 - p) + F(Q, Q/2 + p))/2$. Here the $m^2 - M^2$ term in the denominator can be neglected since in the mass expansion

$$\int \frac{d^4 p}{(2\pi)^4} f(p) (M^2 - m^2) \frac{\Delta_0(p - \frac{Q}{2}) - \Delta_0(p + \frac{Q}{2})}{(2pQ)^2} = 0 \quad (15)$$

because of the odd $p \rightarrow -p$ behaviour of the integrand. Then we only need to expand the difference of Δ 's of the numerator. The numerator can be expanded with respect to Q , when low-momentum mean fields are considered. The leading term of the gradient expansion gives zero, again because of the p -odd integrand.

The first non-zero contribution is therefore

$$I_F(m^2, M^2) = -\frac{Q_\rho}{2} \int \frac{d^4 p}{(2\pi)^4} \frac{f(p)}{2pQ} \frac{\partial}{\partial p_\rho} (\Delta_{m^2}(p) + \Delta_{M^2}(p)). \quad (16)$$

Now, we treat separately the two actual cases: $f = p_\mu p_\nu$ and $f = \text{constant}$. The $f = p_\mu p_\nu$ case contributes to the full expression of $j_\mu^{ind}(Q)$

$$2e^2 A^\nu(Q) Q^\rho \int \frac{d^4 p}{(2\pi)^4} \frac{p_\mu p_\nu}{2pQ} \frac{\partial}{\partial p^\rho} (\Delta_1 + \Delta_2 + \Delta_{m_W^2} - \Delta_{\xi m_W^2}). \quad (17)$$

We introduce the field-strength tensor by $A^\nu Q^\rho = -iF^{\nu\rho} + A^\rho Q^\nu$. The local term proportional to A reads as

$$e^2 A^\rho(Q) \int \frac{d^4 p}{(2\pi)^4} p_\mu \frac{\partial}{\partial p^\rho} (\Delta_1 + \Delta_2 + \Delta_{m_W^2} - \Delta_{\xi m_W^2}). \quad (18)$$

After partial integration the first two terms cancel with $e^2 A_\mu \langle \varphi^2 \rangle$ in the induced current. What remains is a contribution vanishing at zero m_W^2 . In the mass expansion they give

$$j_\mu^{local,1} = -e^2(1 - \xi)m_W^2 A_\mu(Q) \int \frac{d^4 p}{(2\pi)^4} \frac{\partial \Delta_0}{\partial \mathbf{p}^2}. \quad (19)$$

For each Δ_{m^2} the field strength contribution of (17) is rewritten with the help of the relation

$$s^\rho \frac{\partial \Delta}{\partial p^\rho} = -2ps \frac{\partial \Delta}{\partial \mathbf{p}^2} + s_0 \frac{dn(|p_0|)}{dp_0} 2\pi \delta(p^2 - m^2) \quad (20)$$

in the form

$$-2ie^2 \int \frac{d^4 p}{(2\pi)^4} \left(-\frac{p_\mu}{pQ} p_\nu p_\rho F^{\nu\rho}(Q) \frac{\partial \Delta_{m^2}}{\partial \mathbf{p}^2} + F^{\nu 0}(Q) \frac{p_\mu p_\nu}{2pQ} \frac{dn(|p_0|)}{dp_0} 2\pi \delta(p^2 - m^2) \right). \quad (21)$$

The first term drops out because of the antisymmetry of F . In the second term we perform the mass expansion. After adding the different contributions we find

$$j_\mu^{(1)} = 4e^2(-iF^{\nu 0}) \int \frac{d^4 p}{(2\pi)^4} \frac{p_\mu p_\nu}{2pQ} \frac{dn(|p_0|)}{dp_0} \left[2\pi \delta(p^2) - \frac{m_H^2 + m_G^2 + m_W^2}{2} 2\pi \delta'(p^2) \right] \equiv \Pi_{\mu\nu} A^\nu. \quad (22)$$

The first term of this expression is the usual HTL contribution, the second is a mass correction to it. In the mass correction, originally, there was $m_1^2 + m_2^2 + m_W^2 - \xi m_W^2$, but because of $m_2^2 = m_G^2 + \xi m_W^2$ the ξ dependence drops out.

The $f = \text{constant}$ contribution to the induced current appears as

$$-4e^2 m_W^2 A_\mu(Q) Q^\rho \int \frac{d^4 p}{(2\pi)^4} \frac{1}{2pQ} \frac{\partial \Delta_0}{\partial p^\rho}. \quad (23)$$

Since it is already proportional to m_W^2 we have dropped the mass dependence of the integral. Using once more the relation (20) we write for it

$$-4e^2 m_W^2 A_\mu(Q) \int \frac{d^4 p}{(2\pi)^4} \frac{1}{2pQ} \left(-2pQ \frac{\partial \Delta_0}{\partial \mathbf{p}^2} + Q_0 \frac{dn(|p_0|)}{dp_0} 2\pi \delta(p^2) \right). \quad (24)$$

The first term is again local:

$$j_\mu^{local,2} = 4e^2 m_W^2 A_\mu(Q) \int \frac{d^4 p}{(2\pi)^4} \frac{\partial \Delta_0}{\partial \mathbf{p}^2}. \quad (25)$$

The second term reads as

$$j_\mu^{(2)} = -4e^2 m_W^2 Q_0 A_\mu \int \frac{d^4 p}{(2\pi)^4} \frac{1}{2pQ} \frac{dn(|p_0|)}{dp_0} 2\pi \delta(p^2). \quad (26)$$

3.2 Odd contributions

Odd contributions come exclusively from $\langle a_\mu \varphi_1 \rangle$. Using (14) and denoting the current by $j_\mu^{(3)}$ we find

$$j_\mu^{(3)} = 2e^2 A^\nu \int \frac{d^4 p}{(2\pi)^4} (p_\mu Q_\nu + p_\nu Q_\mu) \frac{\Delta_{m_W^2}(p - \frac{Q}{2}) - \Delta_1(p + \frac{Q}{2})}{2pQ + m_W^2 - m_1^2} - \{m_W^2 \rightarrow \xi m_W^2\}. \quad (27)$$

Finally, with the help of (20), performing the mass and external momentum expansion according to the method followed in the previous subsection we arrive at

$$j_\mu^{(3)} = 2e^2(1 - \xi)m_W^2 Q_0 A^\nu \int \frac{d^4 p}{(2\pi)^4} \frac{p_\mu Q_\nu + p_\nu Q_\mu}{(2pQ)^2} \frac{dn(|p_0|)}{dp_0} 2\pi \delta(p^2) \equiv Z_{\mu\nu} A^\nu. \quad (28)$$

3.3 Linear response and physical modes

The full induced current is the sum of the different parts coming from (19), (22), (25), (26) and (28):

$$j_\mu^{ind} = j_\mu^{local} + j_\mu^{(1)} + j_\mu^{(2)} + j_\mu^{(3)}, \quad (29)$$

where $j_\mu^{local} = j_\mu^{local,1} + j_\mu^{local,2}$.

Here $j_\mu^{(1)}$ and $j_\mu^{(2)}$ are ξ -independent, while j_μ^{local} and $j_\mu^{(3)}$ depend on the gauge fixing. For the physical characterization of the system (for instance, damping rates) we have to find the independently evolving modes. In order to do this we decompose the polarization tensor in the tensor basis, appropriate for finite-temperature studies [27]:

$$P_{\mu\nu}^T = -g_{\mu i}(\delta_{ij} - \hat{Q}_i \hat{Q}_j)g_{\nu j}, \quad P_{\mu\nu}^L = -\frac{Q^2}{q^2}u_\mu^T u_\nu^T, \quad P_{\mu\nu}^G = \frac{Q_\mu Q_\nu}{Q^2}, \quad S_{\mu\nu} = \frac{1}{q} (Q_\mu u_\nu^T + u_\mu^T Q_\nu), \quad (30)$$

where $Q = (q_0, \mathbf{q})$, $\mathbf{q}^2 = q^2$ and $u_\mu^T = g_{\mu 0} - Q_\mu q_0 / Q^2$ was used.

Since $\Pi_{\mu\nu}$ of (22) is transverse ($Q^\mu \Pi_{\mu\nu} = 0$), it is the combination of P^T and P^L . Introducing $\Pi_L = \Pi_{00}$ and $\Pi_T = 1/2 P_{\mu\nu}^T \Pi^{\mu\nu}$ we find

$$\Pi = -\frac{Q^2}{q^2} \Pi_L P^L + \Pi_T P^T. \quad (31)$$

The $Z_{\mu\nu}$ term in (28) can be written as a combination of P^G and S . Introducing $Z_G = Z_\mu^\mu$ and $Z_S = (q_0^2 Z_G - Q^2 Z_{00})/(qq_0)$ we find

$$Z = Z_G P^G + Z_S S. \quad (32)$$

Using the completeness relation $P^L + P^T + P^G = g$ the Fourier transform of the EOM (4) has the following form:

$$\left[(Q^2 - R - \Pi_T) P^T + \left(Q^2 - R + \frac{Q^2}{q^2} \Pi_L \right) P^L + \left(\frac{1}{\xi} Q^2 - R - Z_G \right) P^G - Z_S S \right] A = 0, \quad (33)$$

with R defined from $m_W^2 A_\mu + j_\mu^{local} + j_\mu^{(2)} \equiv R A_\mu$.

Since S is not a projector, but mixes the subspaces belonging to P^L and P^G , in this two-dimensional subspace the inverse propagator matrix still has to be diagonalized:

$$\begin{pmatrix} Q^2 - R + \frac{Q^2}{q^2}\Pi_L & -Z_S \\ Z_S & \frac{1}{\xi}Q^2 - R - Z_G \end{pmatrix}. \quad (34)$$

The eigenvalues should be found with one-loop accuracy, which means that terms proportional to the square of one-loop corrections (i.e. Π_L^2 , Z_G^2 and Z_S^2) should be neglected. This, however, implies that the eigenvalues are the diagonal entries. With appropriately rotated projectors in the (P^G, P^L) plane, we can therefore write

$$\left[(Q^2 - R - \Pi_T)P^T + \left(Q^2 - R + \frac{Q^2}{q^2}\Pi_L \right) \tilde{P}^L + \left(\frac{1}{\xi}Q^2 - R - Z_G \right) \tilde{P}^G \right] A = 0, \quad (35)$$

As expected, the transverse modes plus the \tilde{P}_L mode, which might be called longitudinal, can be made independent of the gauge-fixing parameter (see the discussion on the renormalization of R below).

3.4 The infrared separation scale

The p -integrals in all terms of j_μ^{ind} are factorized into a radial and an angular integral. It is well known that the HTL term describes the dynamical screening of the electric fluctuations below the Debye scale, an infrared separation scale (“IR cut-off”) has to be therefore introduced into the radial integration at $p = M = C_M \times eT$. The effective equations one arrives at in this way are to be used for the modes $p \leq M$.

Applying this cut-off to the integrals appearing in the expressions of $R(Q)$ and $\Pi_{\mu\nu}$, one finds

$$\begin{aligned} R &= m_W^2 \left[1 + \frac{3+\xi}{8\pi^2} e^2 \ln \frac{\kappa T}{\Lambda} + \frac{e^2 T}{2\pi^2 M} \frac{q_0}{q} \ln \frac{q_0 + q}{q_0 - q} \right], \\ \Pi_L &= m_D^2 \left(1 - \frac{q_0}{2q} \ln \frac{q_0 + q}{q_0 - q} \right) + \frac{q^2}{Q^2} \frac{e^2 T}{4\pi^2 M} (m_W^2 + m_G^2 + m_H^2), \\ \Pi_T &= m_D^2 \frac{q_0}{2q} \left[\frac{q_0}{q} - \frac{q_0^2 - Q^2}{2q^2} \ln \frac{q_0 + q}{q_0 - q} \right] + \frac{e^2 T}{4\pi^2 M} (m_W^2 + m_G^2 + m_H^2) \frac{q_0}{q} \ln \frac{q_0 + q}{q_0 - q}, \end{aligned} \quad (36)$$

with $\kappa = 2\pi \exp(-\gamma_E)$ and $m_D^2 = e^2 T^2/3 - e^2 MT/\pi^2$. The logarithmical UV divergence in R can be absorbed into the e^2 renormalization. With appropriate renormalization scale $\mu = \kappa T$ the ξ dependence can be made to vanish.

The intermediate IR scale M contributes a term to the Debye mass, which depends linearly on M . It will be cancelled by the linear M -dependence of the self-energies, which shows up in the (one-loop) solution of the classical effective EOM (4) [28, 29, 10, 30].

The mass corrections in (35), proportional to $\bar{\Phi}^2 A^\nu$, should be interpreted as coming from an induced (non-local) $\Phi^2 A^2$ vertex correction. In the subsequent classical time evolution it contributes to the self-energies of both fields. When (perturbatively) combined with the $\sim MT$ terms coming from the tadpoles, it results in a finite contribution of order T^2 , independent of the exact choice of the coefficient C_M in the expression of M . Therefore, these terms can by no means be neglected.

4 Conclusions

In this letter we have investigated the induced current in the effective EOM of the gauge field in the Abelian Higgs model. Beyond the well-known HTL term, we now determined also the subleading $\sim \Phi^2 A_\mu$ corrections in adiabatic approximation ($\Phi = \text{constant}$). The most important property of the corrections is that, just as the leading HTL term, in the high-temperature expansion for the physical degrees of freedom it is independent of the gauge-fixing procedure (after appropriate renormalization).

For consistency we have applied an IR cut-off $M = C_M \times eT$ to the fluctuations, and the effective EOM is valid below this scale. The subsequent 3D time evolution will be insensitive to the accurate choice of M . Partly it is cancelled by the 3D (Rayleigh–Jeans-type) divergences, partly it yields also non-zero, M -independent contributions, when the gauge–scalar vertex corrections are combined with the 3D would-be divergences.

We shortly discuss here the numerical implementation of the effective gauge field equation. The most convenient gauge fixing seems to be the Landau-gauge, where the ξ -dependent mode does not propagate. Then $j_\mu^{(3)}$ can be left out of the discussion and the effective equations of motion are to be solved under the constraint $\partial_\mu A^\mu = 0$. It can be implemented, for example, by solving the equations for A_i and computing A_0 from the constraint.

The non-local induced currents are written in local form with the help of auxiliary fields. With the well-known form of the leading HTL current, we can write in this form also the corrections due to the non-zero scalar background:

$$\begin{aligned} j_i^{(1)}(x) &= m_D^2 \int \frac{d\Omega_v}{4\pi} \left(v_i v_j - \frac{e^2 T}{2\pi^2 M} \frac{m_W^2 + m_G^2 + m_H^2}{m_D^2} \delta_{ij} \right) W_j^{(1)}(x, \mathbf{v}), \\ j_i^{(2)}(x) &= \frac{e^2 m_W^2 T}{\pi^2 M} \int \frac{d\Omega_v}{4\pi} W_i^{(2)}(x, \mathbf{v}), \end{aligned} \quad (37)$$

where the auxiliary fields satisfy

$$(\partial_0 - \mathbf{v}\partial)W_i^{(1)} = F_{0i}, \quad (\partial_0 - \mathbf{v}\partial)W_i^{(2)} = \partial_0 A_i. \quad (38)$$

In the derivation of the expression of $j_\mu^{(1)}$ we have performed a partial p -integration in the part of (22) proportional to $\delta'(p^2)$, and used the fact that, to the accuracy of our calculation, $Q_\nu F^{\nu\mu}(Q) = 0$ can be exploited in the expression of the induced current.

The equations for the scalar field receive mainly local corrections (renormalization and T -dependence of the couplings in the classical equations). However, for consistency also the Φ -derivative of the Φ^2 – A^2 vertex correction established in the present calculation should be introduced.

The logics of the derivation followed in the Abelian Higgs model seem to be robust enough for us to attempt its generalization to the non-Abelian case, which will be the subject of a future study. A numerical study of the corrected EOMs can lead us to a deeper understanding of the reliability of our present views on the high-temperature Higgs models, especially where the sphaleron rate is concerned. We will also learn in more detail of the non-equilibrium aspects of the cosmological onset of the Higgs regime.

Acknowledgement

The authors acknowledge fruitful discussions with P. Petreczky and H. J. de Vega in the early stages of this work. A.J. thanks D. Litim, C. Manuel, G. Moore, A. Rebhan and M. Thoma, for valuable comments and discussions. This research was supported by the Hungarian Science Fund (OTKA).

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